

Hong Kong Mathematics Olympiad (1999 – 2000)

Heat Event (Group)

香港数学竞赛 (1999 – 2000)

初赛项目(团体)

除非特别声明，答案须用数字表达，并化至最简。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 如果  $a$  是  $x^2 + 2x + 3 = 0$  的根，求  $\frac{a^5 + 3a^4 + 3a^3 - a^2}{a^2 + 3}$  的值。

If  $a$  is a root of  $x^2 + 2x + 3 = 0$ , find the value of  $\frac{a^5 + 3a^4 + 3a^3 - a^2}{a^2 + 3}$ .

2. 方程  $(\cos^2 \theta - 1)(2\cos^2 \theta - 1) = 0$  恰有  $n$  个根，其中  $0^\circ < \theta < 360^\circ$ 。求  $n$  的值。

There are exactly  $n$  roots in the equation  $(\cos^2 \theta - 1)(2\cos^2 \theta - 1) = 0$ , where  $0^\circ < \theta < 360^\circ$ . Find the value of  $n$ .

3. 求  $2004^{2006}$  的个位数。

Find the unit digit of  $2004^{2006}$ .

4. 设  $x = |y - m| + |y - 10| + |y - m - 10|$ ，其中  $0 < m < 10$  和  $m \leq y \leq 10$ 。求  $x$  的最小值。

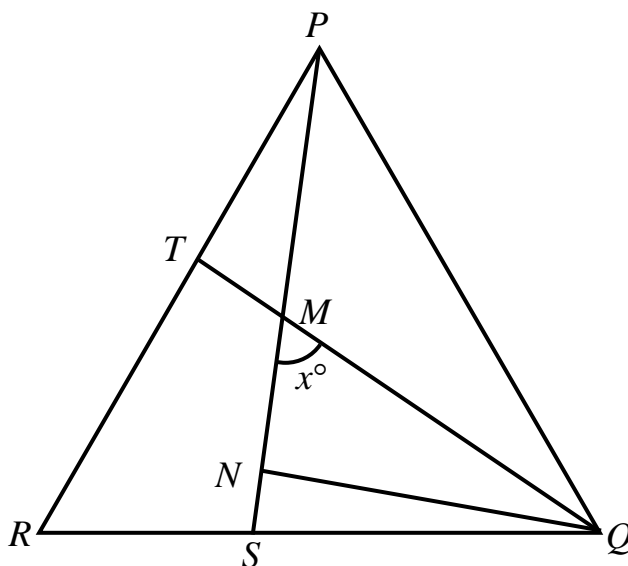
Let  $x = |y - m| + |y - 10| + |y - m - 10|$ , where  $0 < m < 10$  and  $m \leq y \leq 10$ . Find the minimum value of  $x$ .

5. 有 5 个分别标上  $A, B, C, D, E$  的球及 5 个分别标上  $A, B, C, D, E$  的袋，每个袋放一个球。求恰好有 3 个球的标号与袋的标号相同的投放方法总数。

There are 5 balls with labels  $A, B, C, D, E$  respectively and there are 5 pockets with labels  $A, B, C, D, E$  respectively. A ball is put into each pocket. Find the number of ways in which exactly 3 balls have labels that match the labels on the pockets.

6. 如图一,  $\Delta PQR$  为一等边三角形,  $PT=RS$ ;  $PS$ 、 $QT$  相交于  $M$ ;  $QN$  垂直  $PS$  于  $N$ 。设  $\angle QMN = x^\circ$ , 求  $x$  的值。

In Figure 1,  $\Delta PQR$  is an equilateral triangle,  $PT=RS$ ;  $PS$ ,  $QT$  meet at  $M$ ; and  $QN$  is perpendicular to  $PS$  at  $N$ . Let  $\angle QMN = x^\circ$ , find the value of  $x$ .



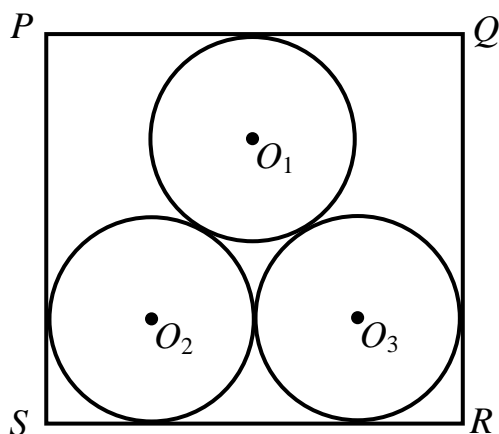
图一

Figure 1

7. 如图二, 已知三等圆互相外切, 且内切于矩形  $PQRS$ , 求  $\frac{QR}{SR}$  的值。

(取  $\sqrt{3} = 1.7$  及答案须准确至二个小数字)

In Figure 2, three equal circles are tangent to each other, and inscribed in rectangle  $PQRS$ , find the value of  $\frac{QR}{SR}$ . (Use  $\sqrt{3} = 1.7$  and give the answer correct to 2 decimal places)



图二

Figure 2

8. 两个正整数之和为 29, 求此两数平方和的最小值。

The sum of two positive integers is 29, find the minimum value of the sum of their squares.

9. 设  $x = \sqrt{3+\sqrt{3}}$  及  $y = \sqrt{3-\sqrt{3}}$ , 求  $x^2(1+y^2) + y^2$  的值。

Let  $x = \sqrt{3+\sqrt{3}}$  and  $y = \sqrt{3-\sqrt{3}}$ , find the value of  $x^2(1+y^2) + y^2$ .

10. 袋内有球 9 个, 分别标上整数 1 到 9。甲从袋中随机地抽出一个球并把它放回, 乙再从同一袋中随机地抽出一个球。把两球上的整数相加, 设  $n$  为该和的个位数字,  $P(n)$  为  $n$  出现的概率。求  $n$  的值使得  $P(n)$  为最大。

There are nine balls in a pocket, each one having an integer label from 1 to 9. A draws a ball randomly from the pocket and puts it back, then B draws a ball randomly from the same pocket. Let  $n$  be the unit digit of the sum of numbers on the two balls drawn by A and B, and  $P(n)$  be the probability of the occurrence of  $n$ . Find the value of  $n$  such that  $P(n)$  is the maximum.